Announcements

Next plans: Feb break, then very short hw due Friday as usual

A few course policies

1. **Regrade requests are due within 1 week of the grade opening on Gradescope**
2. Recall: **Participation** & completion of course eval part of you grade (3%).
    a. Poll Everywhere, with ≥75% credit is full credit
    b. Section participation (after the quiz), missing up to 2 sections is full credit
    c. Course eval response
3. Will drop 2 of your worst quiz scores
4. We **do not drop** any hw scores

Solutions are always posted on Canvas. Please read pinned posts on Ed

prelim solutions will be posted once we open grades

Join by Web **PollEv.com/evatardos772**

How was the prelim?

A. Challenging
B. OK
C. OK, but too long, I ran out of time
D. I found it easy

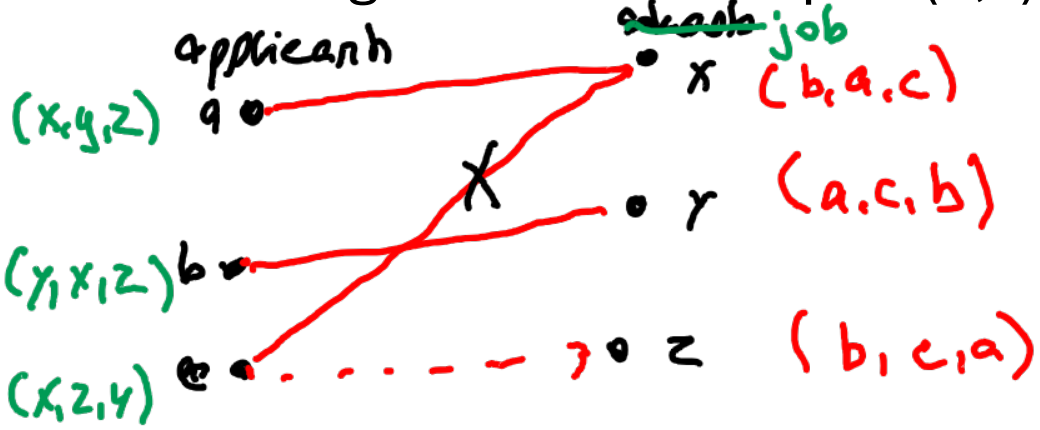
Stable Matching II:

Recall Stable Matching:

two sides: medical residents and hospitals, or students and schools

Both sides order the other side by preference

A matching is unstable if a pair (a,x) prefer each other to their current partner



Gale - Shapley

While ∃ applicant unmatched & job they have not applied to

- select such an applicant
- apply to best job not yet applied to
- jobs: keep best applicant

endwhile

Return current matchings

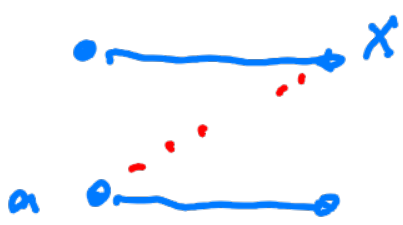
Gale Shapley Algorithm: and properties we have seen

1. n applicants m jobs max nm applications
2. $n = m$ implies output is perfect matching (all applicants assigned to jobs)
3. * jobs once get applicant they have one & it keeps getting better
4. applicants by in decreasing of their preference

Does Gale Shapley produce a Stable solution?

$n = m$

unstable (perfect matching)



if (a,x) not matched
& a prefers x to its assignment
x — " — a — " —

stable = no unstability

Claim: if $n=m$ Gale-Shapley returns stable matching

Proof:

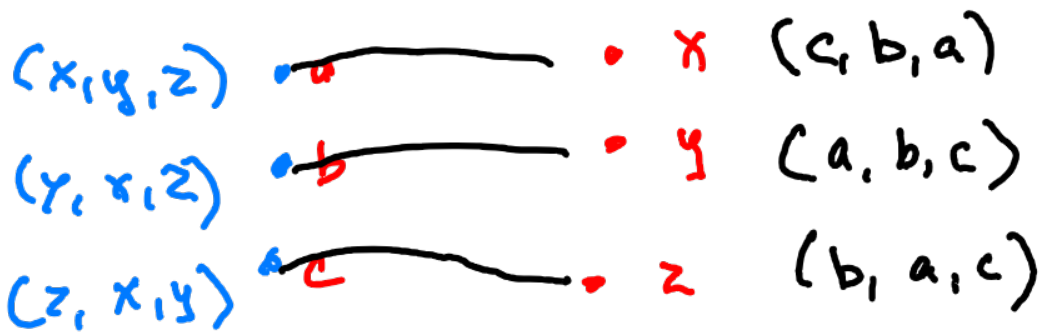
consider pair (a,x)
did a ever apply to job x

not unstable ⇐

if yes ⇒ a got rejected, hence x has better applicant due (*) (5)

if no ⇒ they got accepted at a job they prefer

Which stable matching does Gale-Shapley find?



Does order of making offers change outcome?

Define: best match for applicant a: best on their list where they can be matched in stable solution

best(a) this best match

Theorem All applicant in Gale-Shapley matched to best match

$\Rightarrow$ order of offers does not matter

$\Rightarrow$ (a, best(a)) is matching i.e best(a) $\neq$ best(b)

Note: job may not get their best

The GS outcome best for the proposers (cont.)

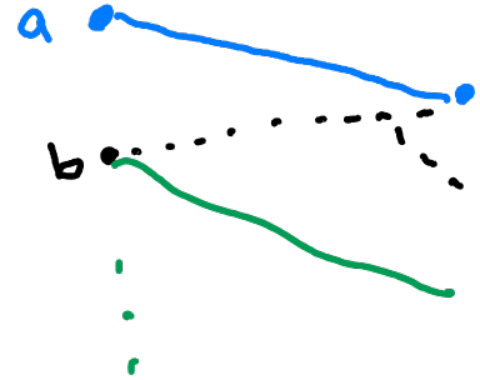
Proof of theorem:

(1) GS outcome for all applicant a, their match $\leq$ best(a)

by contradiction

some applicant a got rejected by $x = best(a)$

consider the first such event



rejected, because other applicant b

best(a)
∃ stable matching
a & best(a) matched

Claim: proposed matching (a, best(a)) cannot be stable

$(b, x) = (b, best(a))$ $\prec$ x prefers b = rejection
b prefers x because no one has been

rejected from their best yet

The GS outcome best for the proposers (cont.)

Claim (also true)

job x worst(x) = applicant they can get is a stable solution that lowest on their list

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All great if applicant rank all options

harder when this possible or viable